

High-Fidelity Teleportation of Independent Qubits

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Abstract

Quantum teleportation is one of the essential primitives of quantum communication. We suggest that any quantum teleportation scheme can be characterized by its efficiency, i.e. how often it succeeds to teleport, its fidelity, i.e. how well the input state is reproduced at the output, and by its insensitivity to cross talk, i.e. how well it rejects an input state that is not intended to teleport. We discuss these criteria for the two teleportation experiments of independent qubits which have been performed thus far. In the first experiment (Nature **390**, 575 (1997)) where the qubit states were various different polarization states of photons, the fidelity of teleportation was as high as 0.80 ± 0.05 thus clearly surpassing the limit of $2/3$ which can, in principle, be obtained by a direct measurement on the qubit and classical communication. This high fidelity is confirmed in our second experiment (Phys. Rev. Lett. **80**, 3891 (1998)), demonstrating entanglement swapping, that is, realizing the teleportation of a qubit which itself is still entangled to another one. This experiment is the only one up to date that demonstrates the teleportation of a genuine unknown quantum state.

1 Introduction

Two of the most fundamental protocols of quantum communication are quantum teleportation and entanglement swapping [1, 2], the teleportation

of an entangled state. With the qubit being the elementary representative of information in the quantum domain, teleportation and entanglement swapping of qubits are essential contributions to any quantum communication toolbox. Thus far there have been two experiments performed [3, 4] on the teleportation of independent qubits. Another experiment [5] demonstrated the quantum teleportation protocol not for an independent qubit but for a qubit that has to be prepared on a specific particle (entangled with another particle). And finally, a fourth experiment [6] demonstrated the quantum teleportation for continuous variables. In the present paper we suggest ways how to characterize the quality of a given teleportation scheme and we discuss specifically the experiments on teleportation of independent qubits [3, 4] from that perspective. We show explicitly that it is important to distinguish teleportation fidelity from teleportation efficiency. That way some criticism which has been raised in the literature [7] turns out to be unjustified [8]. In section 2 we will first briefly review the two experiments concerning teleportation of independent qubits before giving our criteria for experimental quantum teleportation in section 3. In sections 4 and 5 the two experiments will be analyzed in view of the given criteria. Conclusions are drawn in section 6.

2 Experimental Quantum Teleportation of Independent Qubits

In the quantum teleportation experiment presented in Ref. [3] an incoming UV pump-pulse has two opportunities to create pairs of photons (Fig.1). The idea is that on the path from left to right the pulse creates an entangled pair. This is the ancillary entangled pair of the original proposal [1]. One of the ancillaries is passed on to Alice and the other one to Bob. The latter one will obtain the teleported qubit encoded in its polarization. On the return path the pulse again creates a pair of photons where in the original experimental teleportation scheme the fact that the two are entangled was not utilized. In fact, one of these two photons was passed through an adjustable polarizer such defining the state (the initial qubit) to be teleported. This procedure breaks the entanglement for that pair. The second photon of that pair is sent to a trigger detector whose purpose it was to reject all detector events where this second pair was not created. In the experiment the entangled photons, photons 2 and 3 in Fig.1, were produced in the anti-symmetric

state

$$|\Psi^-\rangle_{23} = \frac{1}{\sqrt{2}} (|H\rangle_2|V\rangle_3 - |V\rangle_2|H\rangle_3) , \quad (1)$$

where $|H\rangle$ and $|V\rangle$ represent the horizontally- and vertically-polarized photon state.

The idea of the experiment then is that Alice subjects the photon to be teleported and her ancillary photon to a (partial) Bell-state measurement using a beam-splitter. Observation of a coincidence at the Bell-state analyzer detectors f1 and f2 then informs Alice that her two photons were projected into the anti-symmetric state $|\Psi^-\rangle_{12}$.

This then implies that Bob's photon is projected by Alice's Bell-state measurement onto the original state. This can be seen by assuming that it is the intention of the experiment to be able to teleport the general qubit,

$$|\Psi\rangle_1 = \alpha|H\rangle_1 + \beta|V\rangle_1 , \quad (2)$$

with α and β complex amplitudes satisfying $|\alpha|^2 + |\beta|^2 = 1$. Then the initial state of qubit plus ancillaries is given by the product state

$$|\Psi\rangle_{123} = |\Psi\rangle_1 |\Psi^-\rangle_{23} . \quad (3)$$

Projection of photon 1 and 2 onto the anti-symmetric state yields

$$\langle\Psi^-|_{12}|\Psi_{123}\rangle = -\frac{1}{2}(\alpha|H\rangle_3 + \beta|V\rangle_3) . \quad (4)$$

This indicates that the polarization state determined by the complex amplitude α and β has been transferred from photon 1 to photon 3. The amplitude factor of $1/2$ indicates that only in one out of four cases the result of the Bell-state measurement is the anti-symmetric one [1, 3].

The experimental scheme then simply proceeded in defining various different polarization states using polarizers and wave plates and verifying by polarization measurement that Bob's photon actually had the state adjusted by the polarizers and wave-plates it never saw given that the coincidence between the detectors f1-f2 did indicate a $(|\Psi^-\rangle_{12})$ Bell-state measurement. In order to demonstrate the generality of the scheme it is not enough to just demonstrate the teleportation of the base states $|H\rangle$ and $|V\rangle$, which readily succeeded in experiment, but also to demonstrate superpositions of these states. In the experiment it was decided to demonstrate teleportation both for two real-coefficients superpositions (linear polarization), and for one superposition with imaginary-coefficients representing circular polarization.

The second experiment [4] demonstrated the teleportation of an entangled state [1] by verifying the protocol of entanglement swapping [2]. Experimentally, the essential difference was that in that experiment (Fig.2) the entanglement of the pair created by the pulse upon its return passage was also fully utilized. Therefore, in that experiment there was no polarizer in the path of that photon of the second pair which was sent to Alice's Bell-state analyzer thus not breaking the initial entanglement. This means that the state when two separate pairs were created in the way described reads

$$|\Psi\rangle_{1234} = |\Psi^-\rangle_{14}|\Psi^-\rangle_{23}, \quad (5)$$

which is a product state of two entangled pairs. Observation of a coincidence at the detectors f1-f2 again indicates that photon 1 and 2 have been projected into the anti-symmetric Bell-state which now indicates that the final state is $|\Psi^-\rangle_{34}$. This shows that now the outer two photons 3 and 4 have become entangled. This can be seen as teleportation either of the state of photon 2 over to photon 4 or the state of photon 1 over to photon 3. Those viewpoints are completely equivalent. The remarkable feature of that experiment is that the actually teleported state is a photon state which is not well defined. Since, as is well known, the state of a particle which is maximally entangled to another one has to be described by a maximally mixed density matrix. Indeed, in that experiment neither of the two photons subject to the Bell-state measurement enjoyed a quantum state on its own. They were both maximally mixed. Therefore, what is teleported in such a situation is not the quantum state of the photon but just the way how it relates to the other photon it has been entangled to initially.

In order to demonstrate that teleportation succeeds in that case, it is necessary to show that photons 3 and 4 are now entangled with each other. This can be done by showing that the polarizations of the two photons are always orthogonal irrespective of the detection basis chosen [4].

3 Criteria for Experimental Quantum Teleportation

We will now identify criteria and notions by which the quality of a certain teleportation procedure can be evaluated. This may also serve to a certain extent as a means for comparison of different teleportation procedures. However, it will turn out that it seems impossible to define just a single parameter which would serve to characterize all procedures.

Any quantum teleportation procedure can be characterized by how well it can answer the following questions:

- 1 How well can it teleport any arbitrary quantum state it is intended to teleport? This is the fidelity of teleportation.
- 2 How often does it succeed to teleport, when it is given an input state within the set of states it is designed to teleport? This is the efficiency of teleportation.
- 3 If given a state the scheme is not intended to teleport, how well does it reject such a state? This is the cross-talk rejection efficiency.

But foremost, one has to define the set of states the teleportation procedure should be able to handle. It is of little use to talk about a specific procedure but use the wrong states to characterize its performance. The aim of the experiments presented in Refs. [3, 4] (Innsbruck experiments) has been to teleport with high fidelity a qubit, i.e. a two-dimensional quantum state, given by the polarization state of a single photon. Experiments performed at Caltech addressed the transfer of an infinite dimensional quantum state represented by the continuous quadrature amplitude components of an electro-magnetic field [6]. We want to emphasize, that if one talks about one or the other type of experiments one should use the appropriate states for describing it.

In the following two sections we evaluate the above criteria in detail for the two teleportation schemes realized in Innsbruck, particularly in view of the criticism initially voiced by Braunstein and Kimble [7, 9, 10]. It is explicitly not our intention to criticize the Caltech experiment though it will be obvious from our analysis that the claim voiced by Kimble a number of times that the Caltech experiment is the first bona fide verification of quantum teleportation is unjustified.

4 Teleportation of Single Qubits

Let us now analyze the first Innsbruck teleportation experiment of independent qubits [3]. Since it is the intention of the experiment to be able to teleport the general qubit (Eq. 2) encoded in the polarization state of a single photon, it is require (a) that the scheme is able to teleport any superposition of this form with high fidelity and (b) that the scheme does not teleport anything which is not of this form.

What happens, if the system does not output a single photon carrying the desired qubit? This situation can be treated on the same footing as some absorption process along a communication channel. As it is well known from other applications of single-photon quantum communication, like quantum cryptography or quantum dense coding, this will influence the efficiency of the communication system but does not influence the coherence properties of the remaining photons. This comes from the fact that the possibility of absorption of the single photon, the "carrier" of the qubit, does not alter the qubit itself. After renormalisation of a two-dimensional state, the original state, the qubit, is obtained again without any influence on the teleportation fidelity. The situation changes drastically if one considers the Caltech teleportation experiment of the quadrature amplitude components of an electro-magnetic field. In that case, an absorption of light-quanta changes the amplitudes of the various Fock-states and therefore unavoidably changes the quantum state that is transmitted. Consequently, absorption necessarily decreases the fidelity of the teleportation procedure for continuous variables but not for single qubits.

As explained in section 2 an incoming UV pump pulse has two opportunities to create pairs of photons (Fig. 1). This can happen either on the path from left to right or on the return path. The cases where only one pair is produced can be rejected since only the situations are accepted in which the trigger detector p fires together with both Bell-state analyzer detectors f1 and f2. Also, any cases where more than two pairs are created can safely be ignored because in the experiment the total probability of creating one pair per pulse in the modes actually detected is of the order 10^{-4} , which gives a detection rate of three pairs from a single pump pulse of much less than one per day at the experimental parameters.

What then does a three-fold coincidence p-f1-f2 tell us? There are two possibilities. One is that we actually had a case of teleportation of the initial qubit encoded in photon 1. In the experiment this was demonstrated for the 5 polarizer settings H, V, $+45^\circ$, -45° and R (circular). These settings represent non-orthogonal qubits and altogether cover very different directions on the Poincare sphere what provides a proof that the scheme works for an arbitrary superposition. H and V proved the working of the scheme for the natural basis states defined by the properties of the experimental setup. The $+45^\circ$ and -45° linear polarization states proved the proper operation for coherent superpositions with real probability amplitudes and the R state for imaginary amplitudes. This is sufficient to demonstrate that the scheme will work for any superposition, in contrast to the suggestion by Vaidman [11] that more settings are required for a full proof. The fact that

the transfer of a quantum state worked for non-orthogonal states is a direct indication that entanglement is at the heart of the experiments.

The second case when a p-f1-f2 coincidence can occur is when both photon pairs are created by the pulse on its return trip. Thus, in that case no teleported photon arrives at Bob's station and teleportation did not happen. Yet Alice recorded a coincidence count at her Bell state detector. It has been argued by Braunstein and Kimble that this possibility reduces the fidelity of our teleportation scheme. Yet, as we will show now, it actually is an advantage of our scheme that teleportation did not occur in that case. Indeed, the state behind the polarizer in that case contains two identically prepared photons. Therefore, since according to our protocol, we only wish to teleport qubits encoded in single-photon states, it is an advantage of our scheme that teleportation does not occur. Thus our scheme has a high intrinsic cross-talk rejection efficiency for these cases.

It might be argued that a spurious coincidence trigger at Alice's Bell state analyzer reduces the usefulness of such a teleportation scheme. Yet, all that happens is that Bob in such a case does not receive a teleported photon even as the message he receives from Alice might indicate that. There is no problem with that since he was not supposed to have obtained a teleported photon in that case anyway, as the state given to Alice does not fall within the class of states, namely single-photon qubits, the scheme is intended to work for. That Alice falsely thinks that teleportation worked in that case does not do any harm.

Another problem already discussed in the original publication [3] is the fact that only one of the four Bell states was identified. This simply means that the procedure works in 25% of the situations. Only whenever the state the two photons at the Bell state analyzer were projected into happened to be the anti symmetric one it was identified by a coincidence behind the beam splitter. In the other 75% of the cases teleportation was not performed. Which of the Bell-states is actually observed is independent of the qubit given to Alice! All this means is simply that the efficiency of the scheme was significantly reduced without any influence on the fidelity of the qubit quantum teleportation.

Losses occur anyway in any realistic scheme and it is always necessary in any protocol to provide for these cases by means of some communication between the various participants. In that spirit we emphatically stress that a reduction of the efficiency of the procedure, of the fraction of cases where it actually finished, does not reduce at all the fidelity which describes how well the actually teleported qubit agrees with the original one.

Clearly, even if a teleportation procedure is inefficient in the sense of

rarely teleporting the given qubit, the fidelity of those qubits which are teleported can be very high. This is very different from a scheme which finishes the teleportation procedure very frequently but with low fidelity. We will see below that the experiment on entanglement swapping provides a clear case where the distinction between efficiency and fidelity is obvious.

As evidenced by the final verification of the teleported qubits, that is by a polarization measurement, the measured qubit teleportation fidelity was rather high in the experiment [3]. As can be seen from Fig. 3, it typically was of the order of 0.80. This very clearly surpasses the limit of $2/3$ indicated by the dotted line which at best could have been obtained by Alice performing a polarization measurement on the given photon, informing Bob about the measurement result via classical communication, and by Bob accordingly preparing a photon at his output.

In conclusion, neither the fact that sometimes through false coincidences Alice might think that teleportation occurred nor the fact that only one Bell state could be identified is relevant for the teleportation fidelity.

5 Quantum Teleportation of Entanglement

Our statement that the rather low efficiencies of the first Innsbruck teleportation experiments do by no means influence the fidelity is even more obvious in the second experiment where, in a realisation of entanglement swapping, it was possible to teleport a qubit which is still entangled to another one. Figure 2 indicates the entanglement swapping procedure and Fig. 4 is a schematic drawing of the experimental setup. The main difference to the first experiment simply was that photon 1, whose polarization properties had to be teleported, was not prepared in a well-defined state prior to teleportation, but rather in a measurement on its twin, photon 4, at a time after the Bell state analyzer had registered a coincidence. This, undoubtedly, realises teleportation in a clear quantum situation, since entanglement between two particles that did not share a common origin nor interacted with one another in the past is the very result of the teleportation procedure.

As in the first experiment, here too one has to deal with the case that Alice might have false coincidence counts at her Bell state analyzer together with a count at detector D_4 for photon 4 (see Fig. 4). This again simply indicates that two pairs have been emitted to the left with no photon going to Bob. As above, since it is intended to teleport only single-photon qubits, it is an advantage that teleportation did not occur in this case.

In the entanglement teleportation experiment a linear polarizer in front of the detector of photon 4 is set at various angles (Θ). As a consequence, whenever teleportation succeeds, the photon received by Bob should be orthogonal to the detected polarization state of photon 4 (since both pairs 1 and 4, and 2 and 3 are prepared in the anti-symmetric state Ψ^- , and since this state is also monitored by Alice, photon 3 and 4 will be entangled in exactly this state, too (see section 1 and Ref. [4]).

This can be verified by performing a polarization measurement on photon 3 carrying the teleported polarization properties of photon 1. In the experiment it was decided to register the coincidences between the two polarization measurements on photons 3 and 4 as a function of the relative angle between the two polarizations (the polarization of photon 3 is measured in the $+45^\circ/-45^\circ$ basis using a $\lambda/2$ rotation plate and a polarizing beamsplitter, while the polarization of photon 4 is measured after passing a variable polarizer at angle Θ). This is equivalent to a measurement of two-qubit correlations in a Bell inequality experiment ([12, 13]).

Again, since Alice identified one Bell state only, the coincidence rate is reduced. Yet, clearly, the observed coincidence counts show correlations well above the classical maximum of 50% visibility and will violate a Bell-type inequality as soon as the coincidence fringe visibility surpasses the critical threshold value of 71% [14]. This visibility was actually surpassed in an individual run of the experiment where alignment and stability parameters appear to have been very favorable. This, and the regular visibility of $(65 \pm 2)\%$ (Fig. 5., corresponding to a fidelity of 0.82 ± 0.01) indicate that it will be possible to actually demonstrate a violation of Bell's inequality in the near future.

In the case of entanglement teleportation it is really obvious that it is wrong to use a Fock-state description and to include the vacuum state for those cases where teleportation did not occur in the definition of the teleportation fidelity, as has been suggested by Braunstein and Kimble. To underline their claim, they suggested that, instead of following the teleportation protocol as described above, Bob could simply use randomly polarized photons to obtain the same (or even better) teleportation fidelity. Yet, clearly, if Bob were to follow that procedure, it would never be possible to observe non-classical correlations and to achieve a violation of a Bell-type inequality. Indeed, the observed coincidence count rates ($D_3^- D_4$ and $D_3^+ D_4$) would not even show the sinusoidal variation as function of Θ exhibited in Fig. 5.

6 Concluding Remarks

In this contribution we demonstrated explicitly the high fidelity, of the order of 0.8, achieved in the teleportation experiments first performed in Innsbruck. The measurement of the fidelity of the teleportation is based on a four-fold coincidence detection technique. The detection of Bob's photon (photon 3 in Fig. 1) plays the double role of projecting out onto the single-photon input state and of measuring the overlap of the single-photon input state with the teleported single-photon state. The role of projecting onto a single-photon input state can be omitted if other means of preparing a single photon input state had been used. This is however a technical, though difficult, issue that has nothing to do with the actual quantum teleportation procedure and therefore the teleportation fidelity will be exactly the same in such situations.

Even if, more or less for technical reasons, the efficiency of the experiments discussed above was very low, the data shown cannot be obtained with any classical communication procedure. Moreover, they clearly demonstrate the capability of this teleportation procedure being implemented as a quantum channel for other quantum communication schemes, e.g., for quantum cryptography, and that the bona fide receiver can be quite sure about the fidelity of the teleported qubit.

The fact that the discussion about the Innsbruck experiments has not abated yet gives us the impression that our initial reply ([8]) to the criticism voiced by Braunstein and Kimble ([7]) might have been too succinct and condensed. We hope that our present paper will help clarify the essential points such that the debate can be set to rest.

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Figure 1: Experimental set up for the teleportation of qubits: A pulse of ultraviolet (UV) light passing through a non-linear crystal can create the ancillary pair of entangled photons 2 and 3. After retroreflection during its second passage through the crystal the ultraviolet pulse can create another pair of photons, one of which will be prepared in the initial state of photon 1 to be teleported, the other one serves as a trigger indicating that a photon to be teleported is under way. Alice then looks for coincidences after a beam splitter (BS) where the initial photon and one of the ancillaries are superposed. Bob, after receiving the classical information that Alice obtained a coincidence count in detectors f1 and f2 identifying the $|\Psi^-\rangle_{12}$ Bell-state, knows that his photon 3 is in the initial state of photon 1 which he then can check using polarization analysis with the polarizing beam splitter (PBS) and the detectors d1 and d2. The detector P provides the information that photon 1 is under way.

Figure 2: Principle of teleportation of entanglement, also known as entanglement swapping: Two EPR sources produce two pairs of entangled photons, pair 1-4 and pair 2-3. Two photons, one from each pair (photons 1 and 2) are subjected to a Bell-state measurement (BSM). This results in projecting the other two outgoing photons 3 and 4 onto an entangled state.

Figure 3: Fidelity of teleportation of a qubit encoded in the polarisation of a single-photon state: The overlap of the input qubit (represented by photons linear polarised along (a) 45° and (b) 90°) with the teleported qubit has been determined via a four-fold coincidence technique to be as high as 80%. This very clearly surpasses the limit of $2/3$, indicated by the dotted lines, which at best could have been obtained if Alice and Bob had been restricted to classical communication only.

Figure 4: Experimental setup for entanglement swapping, i.e. teleportation of entanglement: A UV-pulse passing through a non-linear crystal can create pair 2-3 of entangled photons. Photon 2 is directed to the beamsplitter (BS). After reflection, during its second passage through the crystal the UV-pulse can create a second pair 1-4 of entangled photons. Photon 1 will also be directed to the beamsplitter to perform a Bell-state measurement (BSM) of photons 1 and 2. When photons 1 and 2 yield a coincidence click on the two detectors behind the beamsplitter a projecting onto the $|\Psi^-\rangle_{12}$ state takes place. As a consequence photons 3 and 4 will also be projected onto an entangled state. To analyse their entanglement we look at coincidences between detectors $D3^+$ and $D4$, and between detectors $D3^-$ and $D4$, for different polarization angles Θ .

Figure 5: Entanglement verification: Four-fold coincidences, resulting from two-fold coincidence $D3^+D4$ and $D3^-D4$ conditioned on the two-fold coincidences at the Bell state measurement, as function of the polarization angle Θ . The two complementary sine curves with a visibility of 0.65 ± 0.02 demonstrate that photons 3 and 4 are polarisation entangled.